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Spin polarisability of the nucleon at NLO in the chiral expansion

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Abstract

We present a calculation of the fourth-order (NLO) contribution to spin-dependent forward Compton scattering in heavy-baryon chiral perturbation theory. No low-energy constants, except for the anomalous magnetic moments of the nucleon, enter at this order. The fourth-order piece of the spin polarisability of the proton turns out to be almost twice the size of the leading piece, with the opposite sign. This leads to the conclusion that no prediction can currently be made for this quantity.

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Compton scattering from the nucleon has recently been the subject of much work, both experimental and theoretical. For the case of unpolarised protons the experimental amplitude is well determined, and in good agreement with the results of heavy-baryon chiral perturbation theory (HBCPT). However the situation with regard to scattering from polarised targets is less satisfactory, not least because until very recently no direct measurements of polarised Compton scattering had been attempted.

The usual notation for spin-dependent pieces of the forward scattering amplitude for real photons of energy ω and momentum q is

$$\epsilon_1^\mu \Theta_{\mu\nu} \epsilon_2^\nu = i e^2 \omega W^{(1)}(\omega) \boldsymbol{\sigma} \cdot (\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_2) + \ldots \tag{1}$$

From a theoretical perspective there is particular interest in the low-energy limit of the amplitude: $e^2 W^{(1)}(\omega) = 4\pi(f_2(0) + \omega^2\gamma) + \ldots$, where γ is the forward spin-polarisability. The low-energy theorem (LET) of Low, Gell-Mann and Goldberger [1] states that $f_2(0) = -\alpha_{em}\kappa^2/2M_N^2$.

In terms of measurable quantities, the low-energy constants $f_2(0)$ and γ can be obtained from measurements at energies above the threshold for pion production, ω_0, via dispersion relations; that for the polarisability gives

$$\gamma = \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_-(\omega) - \sigma_+(\omega)}{\omega^3} d\omega, \tag{2}$$

where $\sigma_\pm$ are the parallel and antiparallel cross-sections for photon absorption; the related sum rule for $f_2(0)$, due to Gerasimov Drell and Hearn, [2] has the same form except that $1/\omega$ replaces $1/\omega^3$.

Before direct data existed, the relevant cross-sections were estimated from multipole analyses of pion electroproduction experiments [3,4]. These showed significant discrepancies between the LET and the GDH sum rule for the difference of $f_2(0)$ for the proton and neutron, though the sum was in good agreement. Indeed even the sign of the difference was different. More recently, measurements have been made with MAMI at Mainz, for photon energies between 200 and 800 MeV; the range will be extended downward to 140 MeV, and a future experiment at Bonn will extend it upwards to 3 GeV [5]. The preliminary data from MAMI [5] suggest a continuing discrepancy between the LET and the sum rule for the proton, though a smaller one than given by the multipole analysis. The most recent analysis using electroproduction data, which pays particular attention to the threshold region, also reduces the discrepancy somewhat [6].

The MAMI data does not currently go low enough in energy to give a reliable result for the spin polarisability, γ. However electroproduction data have also been used to extract this quantity; Sandorfi *et al.* [4] find $\gamma_p = -1.3 \times 10^{-4}$ fm^4 and $\gamma_n = -0.4 \times 10^{-4}$ fm^4, while the more recent analysis of Drechsel *et al.* [7] gives a rather smaller value of $\gamma_p = -0.6 \times 10^{-4}$ fm^4. (We shall use units of 10^{-4} fm^4 for polarisabilities from now on.)

The spin polarisability has also been calculated in the framework of HBCPT: at lowest (third) order in the chiral expansion this gives $\gamma = \alpha_{em} g_A^2/(24\pi^2 f_\pi^2 m_\pi^2) = 4.54$ for both proton and neutron, where the entire contribution comes from πN loops. The effect of the Δ enters in counter-terms at fifth order in standard HBCPT, and has been estimated to be so large as to change the sign [8]. The calculation has also been done in an extension

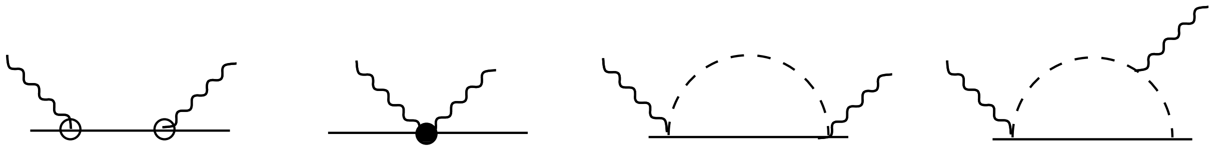

Fig. 1: Diagrams which contribute to spin-dependent Compton scattering in the $\epsilon \cdot v = 0$ gauge at LO. The open circles are vertices from $\mathcal{L}^{(2)}$ and the solid dot is a vertex from $\mathcal{L}^{(3)}$.

of HBCPT with an explicit Δ by Hemmert *et al.* [9]. They find that the principal effect is from the Δ pole, which contributes -2.4, with the effect of $\pi\Delta$ loops being small, -0.2. Clearly the next most important contribution is likely to be the fourth-order πN piece, and this is the result which is presented here. The effects of the Δ at NLO involve unknown parameters; we might hope that the loop pieces at least will be small.

To calculate the spin-dependent forward scattering amplitude, we work in the gauge $A_0 = 0$, or in the language of HBCPT, $v \cdot \epsilon = 0$, where v^μ is the unit vector which defines the nucleon rest frame. The amplitude of Eq. (1) becomes

$$\epsilon_1^\mu \Theta_{\mu\nu} \epsilon_2^\nu = 2ie^2 v \cdot q W^{(1)}(v \cdot q) \epsilon_{\alpha\beta\gamma\delta} v^\alpha \epsilon_1^\beta \epsilon_2^\gamma S^\delta + \dots \tag{3}$$

where S^μ is the spin operator, which obeys $v \cdot S = 0$. The LET states that $W^{(1)}(0) = -e^2\kappa^2/2M_N^2$, where κ is the anomalous magnetic moment of the nucleon. At leading (third) order this is satisfied, with κ replaced by its bare value, by the combination of the Born terms and the seagull diagram, which has a fixed coefficient in the third-order Lagrangian [8]. The loop diagrams of figure 1 have contributions of order ω which cancel and so do not affect the LET. At order ω^3 they give the result quoted above for the polarisability. Note that in this gauge there is no lowest-order coupling of a photon to a nucleon; the coupling comes in only at second order. The Feynman vertex consists of two pieces, one proportional to the charge current and one to the magnetic moment:

$$\frac{ie}{2M}\Big(Q\epsilon \cdot (p_1 + p_2) + 2(Q + \kappa)[S \cdot \epsilon, S \cdot q]\Big). \tag{4}$$

This and all other vertices are taken from the review of Bernard *et al.* [10]. The two pieces contribute in different diagrams; the first is represented by a solid dot and the second by an open circle in figures 1 and 2.

At NLO, the diagrams which contribute are given in figure 2. There can be no seagulls at this order; since $W^{(1)}(\omega)$ is of first chiral order and is even in ω (two powers of e and one of ω having been pulled out of the amplitude in its definition), it will have an expansion of the form $am_\pi + b\omega^2/m_\pi + \dots$. These non-analytic powers of m_π^2 cannot be present in the basic couplings in the Lagrangian, but can only be generated from loops. It follows that there are no undetermined low-energy constants in the final amplitude.

The insertion on the nucleon propagator of figures 2a and b needs some explanation. Denoting the external nucleon residual momentum by p, the energy $v \cdot p$ starts at second chiral order with the mass shift and kinetic energy. In contrast the space components of p and all components of the loop momentum l and the photon momentum are first order.

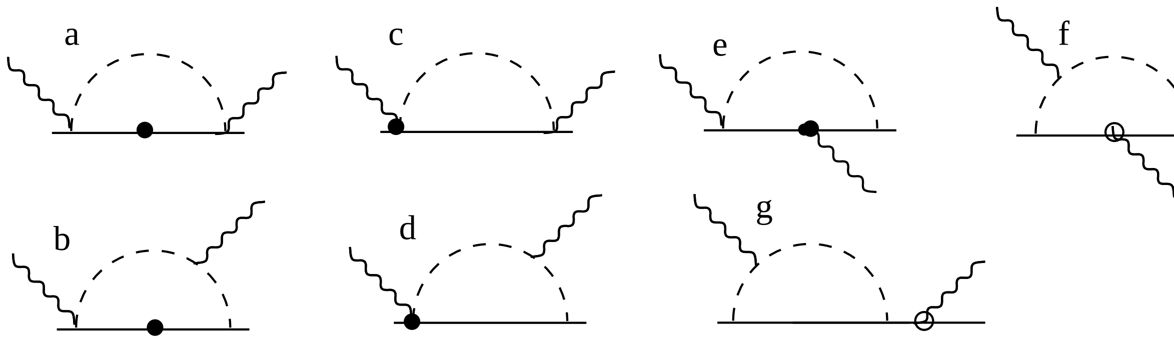

Fig. 2: Diagrams which contribute to spin-dependent forward Compton scattering in the $\epsilon \cdot v = 0$ gauge at NLO. The dots and circles are vertices from $\mathcal{L}^{(2)}$, the solid and open dots at the photon-nucleon vertices representing the couplings proportional to the charge current and magnetic moment respectively.

The propagator with an insertion consists of both the second term in the expansion of the lowest-order propagator, $i/(v \cdot l + v \cdot p)$, in powers of $v \cdot p/v \cdot l$, and also the insertions from $\mathcal{L}^{(2)}$. The second-order mass shift and external kinetic energy cancel between the two to leave, in the rest frame, just the second order propagator given in appendix A of ref. [10].

The contributions of the various diagrams from figure 2 are given in table 1. The final result has the following form:

$$W_4^{(1)}(w) = \frac{g_A^2 m_\pi}{48\pi^2 f_\pi^2 M_N x^2} \left(A + Bx \arccos^2(-x) + \frac{(C + Dx^2 + Ex^4)\arccos(-x)}{\sqrt{1-x^2}} \right) + (x \to -x) \tag{5}$$

where $x = \omega/m_\pi$ and the constants $A - E$ are given below:

$$A = -\pi(1 - \kappa_v + (2 + \kappa_s)\tau_3) \tag{6}$$
$$B = 3\kappa_s\tau_3/2 \tag{7}$$
$$C = 2(1 - \kappa_v + (2 + \kappa_s)\tau_3) \tag{8}$$
$$D = -1 + 4\kappa_v - (2 + \kappa_s)\tau_3) \tag{9}$$
$$E = -(4 + 2\kappa_v + (2 + \kappa_s)\tau_3) \tag{10}$$

It is worth pointing out that the Born-like contribution 2g, like all the two-particle irreducible loop diagrams, is perfectly finite as $\omega \to 0$, as the momentum-dependence of the magnetic coupling cancels the $1/\omega$ of the propagator. Thus the expression in Eq. (5) is also finite, despite superficial appearances.

The total contribution at order ω^0 is

$$W_4^{(1)}(0) = \frac{g_A^2 m_\pi}{16\pi M_N f_\pi^2}(\kappa_v + \kappa_s\tau_3), \tag{11}$$

which, since the one-loop contribution to κ is $\delta\kappa_v = -g_A^2 m_\pi M_N/4\pi f_\pi^2$, can be seen to be exactly the correction to the leading contribution required to satisfy the LET.

Diagram	isospin	$O(\omega)$	$O(\omega^3)$	full function: odd piece only
a	1	3	$-5/2$	$-(2\omega^2-m^2)\partial J_0(\omega)/\partial\omega - 2\omega J_0(\omega)$
b	1	-5	$7/4$	$-4\Big(\partial J_2(\omega)/\partial\omega + m^2(J_2'(\omega)-J_2'(0))/\omega\Big)$
c	$1-\tau_3$	1	$-1/4$	$2(J_2(\omega)-J_2(0))/\omega$
d	τ_3	-2	$1/3$	$4\omega\int_0^1 J_2'(x\omega)dx$
e	τ_3	2	-1	$-2\omega J_0(\omega)$
f	$1+\kappa_v-(1+\kappa_s)\tau_3$	0	$-1/12$	$2\omega\int_0^1(1-2x)J_2'(x\omega)dx$
g	$1+\kappa_v+(1+\kappa_s)\tau_3$	1	$-1/6$	$-2\omega\int_0^1 J_2'(x\omega)dx$

Table 1: Contributions to $2\omega W^{(1)}(\omega)$ from the diagrams of figure 2. All three columns are multiplied by a common factor of $g_A^2/(M_N f^2)$; the coefficients of ω and ω^3 have an extra factor of $m_\pi/8\pi$. The J_i are defined in ref. [10]; a prime denotes differentiation with respect to m^2.

The polarisability to NLO is

$$\gamma = \frac{\alpha_{em} g_A^2}{24\pi^2 f_\pi^2 m_\pi^2}\left[1 - \frac{\pi m_\pi}{8M_N}(15+3\kappa_v+(6+\kappa_s)\tau_3)\right]. \qquad (12)$$

Although this has a factor of m_π/M_N compared with the leading piece, the numerical coefficient is large. Using the physical values of the masses and anomalous magnetic moments gives $\gamma = 4.5-(6.9+1.6\,\tau_3)$. The NLO contributions are disappointingly large, and call the convergence of the expansion into question.

When this work was nearly complete, a preprint on spin-dependent Compton scattering at NLO was sent to the archives by Ji *et al.* [11]. They chose not to work in the $v\cdot\epsilon=0$ gauge, since it is not convenient for the extension to virtual photons which forms their main interest. The price of this is many more diagrams to be evaluated, and two extra Lorentz structures appear multiplying $W^{(1)}$ (namely $S\cdot(\epsilon_1\times q)v\cdot\epsilon_2-(\epsilon_1\leftrightarrow\epsilon_2)$). Some of these diagrams have poles at $\omega=0$. However Ji and coworkers have not included diagrams which contribute only pole terms [12]. As a result their Eq. (10) is incomplete, but after subtracting the pole it agrees with our expression Eq. (5); in particular their expression for γ agrees with ours.

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